Modern Physics Letters A

CHIRAL N=1 4D ORIENTIFOLDS WITH D-BRANES AT ANGLES

GABRIELE HONECKER

Departamento de Física Teórica C-XI and Instituto de Física Teórica C-XVI,
Universidad Autónoma de Madrid, Cantoblanco, 28049 Madrid, Spain
gabriele@th.physik.uni-bonn.de

Received 15 June 2004

D6-branes intersecting at angles allow for phenomenologically appealing constructions of four dimensional string theory vacua. While it is straightforward to obtain non-supersymmetric realizations of the standard model, supersymmetric and stable models with three generations and no exotic chiral matter require more involved orbifold constructions. The $T^6/(\mathbb{Z}_4 \times \mathbb{Z}_2 \times \Omega\mathcal{R})$ case is discussed in detail. Other orbifolds including fractional D6-branes are treated briefly.

Keywords: D-branes, Supersymmetry, Superstring Vacua , String Phenomenology.

PACS Nos.: 11.25.Mj, 12.60.Jv, 11.25.Uv.

1. Introduction

Intersecting D6-branes at angles[1,2,3,4] (for a more complete list of references see[5,6,7,8])[a] provide a powerful framework for model building in type II superstring backgrounds. While compactifications on generic Calabi-Yau manifolds only allow to compute the RR tadpoles and the chiral spectrum, in toroidal and orbifold backgrounds conformal field theory methods facilitate the computation of the complete spectrum, and interactions have a geometric interpretation along the compact dimensions[4]. The Yukawa[13,14,15,16,17,18] and n-point couplings[19], proton decay[20], flavor changing neutral currents[21,22], threshold corrections[23] as well as the tree-level Kähler- and superpotential[18] can be explicitly computed in this framework.

Supersymmetric D6-branes at angles arise naturally in four dimensional orbifold/orientifold compactifications of type IIA string theory[24,25,26,27], [b] if the worldsheet parity Ω is accompanied by an involution $\mathcal{R}$ on the six compact dimensions, i.e. $\mathcal{R}^2 = 1$, which leaves half of the internal directions invariant. The fix loci of $\mathcal{R}\theta^k$, where θ generates the orbifold background, support O6-planes extended along $\mathbb{R}^{1,3}$ plus three compact dimensions, whose charge under the RR 7-form is

[a]For works in the T-dual picture with D9-branes and magnetic backgrounds see e.g. [9,10,11,12].
[b]For the first chiral supersymmetric spectra in this framework see also [39,40].

canceled by the addition of D6-branes also wrapping $\mathbb{R}^{1,3}$ and three compact dimensions. In general, the D6-branes do not wrap the $\mathcal{R}$ invariant cycles. For $\mathcal{R}$ to be a symmetry, therefore, brane images have to be included. If the O6-planes wrap $\mathbb{R}^{1,3} \times \Pi_{O6}$, the D6$_a$-branes extend along $\mathbb{R}^{1,3} \times \Pi_a$ and their $\mathcal{R}$ images along $\mathbb{R}^{1,3} \times \Pi_{a'}$ with $\Pi_{a'} \equiv \mathcal{R}(\Pi_a)$ and $\Pi_{O6}, \Pi_a, \Pi_{a'}$ denoting compact 3-cycles. The condition for the cancellation of the RR 7-form charge reads

$$\sum_a \left(\int_{\mathbb{R}^{1,3}\times\Pi_a} C_7 + \int_{\mathbb{R}^{1,3}\times\Pi_{a'}} C_7 \right) + Q_{O6} \int_{\mathbb{R}^{1,3}\times\Pi_{O6}} C_7 = 0. \tag{1}$$

Including the possibility of N_a identical D6$_a$-branes and inserting the value of the charge of the O6-plane $Q_{O6} = -4$ leads then to the following condition on the compact 3-cycles

$$\sum_a N_a \left(\Pi_a + \Pi_{a'}\right) - 4\Pi_{O6} = 0, \tag{2}$$

i.e. the cancellation of RR tadpoles is the Poincaré dual of the requirement of an overall vanishing class of homology 3-cycles.

The chiral spectrum as well as the RR tadpole cancellation can be expressed purely in terms of the topological setting along the compact dimensions. The intersection number $\Pi_a \circ \Pi_b$ between two kinds of D6$_a$- and D6$_b$-branes counts the net number of chiral bifundamental representations of the gauge groups $U(N_a) \times U(N_b)$ involved. These bifundamental states are located at the pointlike singularity arising at the intersection of the 3-cycles along the compact dimensions. A negative intersection number signals the existence of the complex conjugate representation.[2] The generic chiral spectrum is listed in table 1.

Table 1. Generic chiral spectrum

multiplicity	rep.
$\Pi_a \circ \Pi_b$	$(\mathbf{N}_a, \overline{\mathbf{N}}_b)$
$\Pi_a \circ \Pi_{b'}$	$(\mathbf{N}_a, \mathbf{N}_b)$
$\frac{1}{2}\left(\Pi_a \circ \Pi_{a'} - \Pi_a \circ \Pi_{O6}\right)$	$\mathbf{Sym}_a$
$\frac{1}{2}\left(\Pi_a \circ \Pi_{a'} + \Pi_a \circ \Pi_{O6}\right)$	$\mathbf{Anti}_a$

In generic compactifications with N_a D6$_a$-branes wrapping cycles $\Pi_a \neq \Pi_{a'}$, unitary gauge groups $U(N_a)$ arise which decompose then as $SU(N_a) \times U(1)_a$. For factorizable cycles with $\Pi_a = \Pi_{a'}$, additional open string states become massless, and N_a branes plus their $\mathcal{R}$ images support the gauge group $Sp(2N_a)$ or $SO(2N_a)$. The cubic non-Abelian $SU(N_a)^3$ gauge anomalies vanish automatically upon RR tadpole cancellation (2), whereas mixed anomalies $SU(N_a)^2 - U(1)_b$ and $U(1)_a^2 - U(1)_b$ are in general present. The anomalous $U(1)$ factors acquire a mass through the generalized Green Schwarz mechanism. The relevant couplings to four dimensional RR scalars and their Hodge dual 2-forms are computed from the geometry of the compact space as follows.[28,29,30] Instead of expressing the 3-cycles in terms of an

arbitrary basis, two sets of linearly independent cycles $\Pi_i^{\pm}$ can be chosen such that $\mathcal{R}\left(\Pi_i^{\pm}\right) = \pm\Pi_i^{\pm}$ with $\Pi_i^{+} \circ \Pi_j^{-} = c\delta_{ij}$ and c a model dependent constant. The four dimensional RR scalars and 2-forms then arise as the pullbacks of the Ω even RR 3-form over $\mathcal{R}$ even 3-cycles and the Ω odd 5-form over $\mathcal{R}$ odd 3-cycles,

$$\phi_i = (4\pi^2\alpha')^{-3/2} \int_{\Pi_i^+} {}^{(10)}C_3, \qquad B_2^i = (4\pi^2\alpha')^{-3/2} \int_{\Pi_j^-} {}^{(10)}C_5. \tag{3}$$

A general 3-cycle and its $\mathcal{R}$ image can be expanded in terms of its $\mathcal{R}$ even and odd components,

$$\Pi_a = \sum_i \left(r_a^i \Pi_i^+ + s_a^i \Pi_i^-\right), \qquad \Pi_{a'} = \sum_i \left(r_a^i \Pi_i^+ - s_a^i \Pi_i^-\right). \tag{4}$$

The expansion coefficients r_a^i, s_a^i of the 3-cycles reduce in the effective four dimensional theory to the coefficients of the Green Schwarz couplings,

$$\sum_i 2r_b^i \int_{\mathbb{R}^{1,3}} \phi_i \,\mathrm{tr}F_b \wedge F_b, \qquad N_a \sum_i 2s_a^i \int_{\mathbb{R}^{1,3}} B_2^i \wedge \mathrm{tr}F_a. \tag{5}$$

The latter coupling induces a mass of the Abelian gauge field involved. Therefore, the massless $U(1)$ factors $Q = \sum_a x_a Q_a$ are those with a vanishing coupling to the 2-forms,

$$\sum_a x_a N_a \vec{s}_a = 0. \tag{6}$$

2. The $T^6/(\mathbb{Z}_4 \times \mathbb{Z}_2 \times \Omega\mathcal{R})$ Orientifold

2.1. *Geometry, supersymmetry and RR tadpole cancellation*

The orbifold $T^6/(\mathbb{Z}_4 \times \mathbb{Z}_2)$ has two generators,

$$\begin{aligned} \Theta: &\qquad (z^1, z^2, z^3) \to (iz^1, -iz^2, z^3), \\ \omega: &\qquad (z^1, z^2, z^3) \to (z^1, -z^2, -z^3), \end{aligned}$$

where $z^k = x^{2+2k} + ix^{3+2k}$ label the internal complex coordinates and $x^0, \dots, x^3$ are the non-compact ones. The involution which accompanies the worldsheet parity is chosen to be

$$\mathcal{R}: z^k \longrightarrow \overline{z}^{\overline{k}}, \qquad k = 1, 2, 3. \tag{7}$$

For consistency of the compactification, the orbifold lattice has to be invariant under this involution. The square tori T_1^2, T_2^2 required by the $\mathbb{Z}_4$ symmetry have two possible orientations **A** and **B** w.r.t. the $\mathcal{R}$ invariant axes x^{2k+2}. The geometry is depicted in figure 1. The complex structure on T_3^2 is not fixed by the $\mathbb{Z}_2$ symmetry, and the torus can be either rectangular with the axes along x^8 and x^9 or tilted as depicted.[31]

The Hodge numbers of this orbifold are $h_{1,1} = 61, h_{2,1} = 1$.[32] This means, that $b_3 = 2 + 2h_{2,1} = 4$ linearly independent 3-cycles exist. As in the $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$

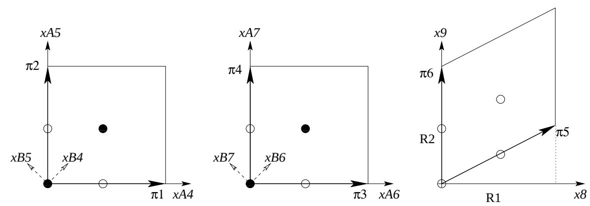

Fig. 1. Geometric set-up of the $T^6/(\mathbb{Z}_4 \times \mathbb{Z}_2 \times \Omega\mathcal{R})$ orientifold. On $T_1^2 \times T_2^2$, the square tori can have either orientation **A** or **B** w.r.t. the $\mathcal{R}$ invariant axis x^{2+2k} $(k = 1, 2)$. The torus T_3^2 can be either untilted or tilted as depicted. Empty circles denote $\mathbb{Z}_2$ fixed points while filled circles correspond to points fixed under $\mathbb{Z}_4$. For further details see the paragraph below eq. (11).

case, all 3-cycles are inherited from the underlying torus and are given by orbifold invariant quantities, e.g. $\rho_1' = (\sum_{k=0}^3 \Theta^k)(\sum_{l=0}^1 \omega^l)\pi_{135}$ with $\pi_{135} \equiv \pi_1 \otimes \pi_3 \otimes \pi_5$. A convenient choice of linearly independent 3-cycles is

$$\rho_1' = 4\,(\pi_{135} - \pi_{245})\,, \qquad \rho_3' = 4\,(\pi_{136} - \pi_{246})\,,$$
$$\rho_2' = 4\,(\pi_{235} + \pi_{145})\,, \qquad \rho_4' = 4\,(\pi_{236} + \pi_{146})\,,$$

where the factor 4 arises from the invariance of any cycle under Θ^2 and ω, and e.g. $\Theta(\pi_{135}) = -\pi_{245}$. As in the $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ case (which has eight independent 3-cycles), the unimodular basis is formed by the cycles $\rho_i \equiv \frac{1}{2}\rho_i'$ with intersection matrix

$$I^{\mathbb{Z}_4\times\mathbb{Z}_2} = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}. \tag{8}$$

Any 3-cycle can be expressed as a linear combination of the ρ_i. The coefficients of the factorizable 3-cycles arise as follows. A factorizable cycle a and its $\mathbb{Z}_4$ image (Θa) are specified by the wrapping numbers (n_k^a, m_k^a) along the 1-cycles (π_{2k-1}, π_{2k}),

$$\begin{aligned} a: &\qquad (n_1^a\pi_1 + m_1^a\pi_2) \otimes (n_2^a\pi_3 + m_2^a\pi_4) \otimes (n_3^a\pi_5 + m_3^a\pi_6)\,, \\ (\Theta a): &\qquad (-m_1^a\pi_1 + n_1^a\pi_2) \otimes (m_2^a\pi_3 - n_2^a\pi_4) \otimes (n_3^a\pi_5 + m_3^a\pi_6)\,. \end{aligned} \tag{9}$$

The homology 3-cycle wrapped by a factorizable D6$_a$-brane therefore is

$$\Pi_a = Y_a n_3^a \rho_1 + Z_a n_3^a \rho_2 + Y_a m_3^a \rho_3 + Z_a m_3^a \rho_4, \tag{10}$$

where the coefficients are $\mathbb{Z}_4$ invariant combinations of the wrapping numbers,

$$Y_a = (n_1^a n_2^a - m_1^a m_2^a)\,, \qquad Z_a = (n_1^a m_2^a + m_1^a n_2^a)\,. \tag{11}$$

The $\mathcal{R}$ image cycle $\Pi_{a'}$ differs for the six inequivalent choices of lattice orientations: on $T_1^2 \times T_2^2$, these are **AA**, **AB** and **BB**; on T_3^2, a rectangular torus **a** with arbitrary ratio of radii $r \equiv R_2/R_1$ has the $\mathcal{R}$ invariant axis π_5, and on a tilted torus **b** with r arbitrary $(2\pi_5 - \pi_6)$ is $\mathcal{R}$ invariant. The latter two choices can be compactly treated by defining $b = 0, \frac{1}{2}$ for the rectangular and tilted torus, respectively. The

specific choices $r = \frac{1}{1-b}$ correspond to square tori with orientations **A** and **B** for $b = 0, \frac{1}{2}$, respectively. Observe that the choice of basis of the **B** orientation on T_3^2 differs from those on $T_1^2 \times T_2^2$.

The $\mathcal{R}$ images of the basis follow from tensoring the results of the 1-cycles, in detail $\pi_{2k-1} \stackrel{\mathcal{R}}{\to} \pi_{2k-1}$, $\pi_{2k} \stackrel{\mathcal{R}}{\to} -\pi_{2k}$ per **A** and $\pi_{2k-1} \stackrel{\mathcal{R}}{\leftrightarrow} \pi_{2k}$ per **B** torus $(k = 1, 2)$ and $\pi_5 \stackrel{\mathcal{R}}{\to} \pi_5 - (2b)\pi_6$, $\pi_6 \stackrel{\mathcal{R}}{\to} -\pi_6$ on T_3^2. The result is listed in table 2.

Table 2. $\mathcal{R}$ images of cycles for $T^6/(\mathbb{Z}_4 \times \mathbb{Z}_2 \times \Omega\mathcal{R})$

lattice	$\mathcal{R} : \rho_1$	$\mathcal{R} : \rho_2$	$\mathcal{R} : \rho_3$	$\mathcal{R} : \rho_4$
AAa/b	$\rho_1 - (2b)\rho_3$	$-\rho_2 + (2b)\rho_4$	$-\rho_3$	ρ_4
ABa/b	$\rho_2 - (2b)\rho_4$	$\rho_1 - (2b)\rho_3$	$-\rho_4$	$-\rho_3$
BBa/b	$-\rho_1 + (2b)\rho_3$	$\rho_2 - (2b)\rho_4$	ρ_3	$-\rho_4$

The $\mathcal{R}$ image of the cycle Π_a for the different lattice choices is given by

$$\Pi_{a'} = \begin{cases} Y_a n_3^a \rho_1 - Z_a n_3^a \rho_2 - Y_a \left[m_3^a + (2b)n_3^a\right] \rho_3 + Z_a \left[m_3^a + (2b)n_3^a\right] \rho_4 & \mathbf{AAa/b}, \\ Z_a n_3^a \rho_1 + Y_a n_3^a \rho_2 - Z_a \left[m_3^a + (2b)n_3^a\right] \rho_3 - Y_a \left[m_3^a + (2b)n_3^a\right] \rho_4 & \mathbf{ABa/b}, \\ -Y_a n_3^a \rho_1 + Z_a n_3^a \rho_2 + Y_a \left[m_3^a + (2b)n_3^a\right] \rho_3 - Z_a \left[m_3^a + (2b)n_3^a\right] \rho_4 & \mathbf{BBa/b}. \end{cases} \tag{12}$$

An arbitrary cycle Π_a at non-trivial angles w.r.t. the $\mathcal{R}$ invariant axis on a generic T_3^2 does not preserve supersymmetry. The supersymmetry condition $\sum_{k=1}^3 \pi\tilde{\varphi}_k^a = 0$,[1] — where $\pi\tilde{\varphi}_k^a = \pi(\varphi_k^a - \frac{c}{4})$ ($c = 0, 1$ for **A**, **B**, respectively, and $k = 1, 2$) is the angle w.r.t. the $\mathcal{R}$ invariant plane on T_k^2 and $\pi\varphi_k^a$ is the angle w.r.t. π_{2k-1} — is cumbersome for computational purposes and more conveniently replaced by two conditions on the wrapping numbers n_3^a, m_3^a and coefficients Y_a, Z_a by using the fact that a vanishing sum over angles implies

$$\sum_{i=1}^3 \tan\pi\tilde{\varphi}_i^a = \prod_{i=1}^3 \tan\pi\tilde{\varphi}_i^a. \tag{13}$$

Due to

$$T_i^2 : i = 1, 2 \qquad \tan\pi\varphi_i^a = \frac{m_i^a}{n_i^a}, \qquad\qquad T_3^2 : \qquad \tan\pi\tilde{\varphi}_3^a = \frac{M_3^a}{n_3^a} r, \tag{14}$$

with $M_3^a \equiv m_3^a + b n_3^a$, the necessary supersymmetry condition in terms of wrapping numbers is

$$\begin{aligned} &\mathbf{AAa/b} \qquad && Z_a n_3^a + Y_a M_3^a r = 0, \\ &\mathbf{ABa/b} \qquad && (Z_a - Y_a)\, n_3^a + (Y_a + Z_a)\, M_3^a r = 0, \\ &\mathbf{BBa/b} \qquad && -Y_a n_3^a + Z_a M_3^a r = 0. \end{aligned} \tag{15}$$

Since the tangent is defined modulo π, (15) does not distinguish between D6-branes and anti-D6-branes. The sufficient condition for supersymmetric D6$_a$-branes is the correct combination of signs for (Y_a, Z_a) and (n_3^a, M_3^a) such that all contributions to the RR tadpoles are positive, see eq. (16) below.

The O6-planes lie on the fixed planes of $\mathcal{R}\Theta^k\omega^l$ and are explicitly given in terms of wrapping numbers and 3-cycles in table 3. Since e.g. on an **A** torus, two parallel vertical or horizontal planes passing through the origin and displaced by $\frac{1}{2}\pi_{2k-1}$ or $\frac{1}{2}\pi_{2k}$ are invariant under $\mathcal{R}\Theta^{2k}\omega^l$, in the last column the number of parallel O6-planes is listed. The over-all cycle Π_{O6} is the sum over all possible fixed planes weighted by their multiplicities.

Table 3. O6-planes for $T^6/(\mathbb{Z}_4 \times \mathbb{Z}_2 \times \Omega\mathcal{R})$

lattice	$(n_1, m_1; n_2, m_2; n_3, m_3)$	Y	Z	cycle	$T_1 \times T_2 \times T_3$
AAa/b	$(1,0;1,0;\frac{1}{1-b},\frac{-b}{1-b})$	1	0	$\frac{1}{1-b}[\rho_1 - b\rho_3]$	$2 \times 2 \times 2(1-b)$
	$(1,1;1,-1;\frac{1}{1-b},\frac{-b}{1-b})$	2	0	$\frac{2}{1-b}[\rho_1 - b\rho_3]$	$1 \times 1 \times 2(1-b)$
	$(1,0;0,1;0,-1)$	0	1	$-\rho_4$	$2 \times 2 \times 2(1-b)$
	$(1,1;1,1;0,-1)$	0	2	$-2\rho_4$	$1 \times 1 \times 2(1-b)$
ABa/b	$(1,0;1,1;\frac{1}{1-b},\frac{-b}{1-b})$	1	1	$\frac{1}{1-b}[\rho_1 + \rho_2 - b(\rho_3 + \rho_4)]$	$2 \times 1 \times 2(1-b)$
	$(1,1;1,0;\frac{1}{1-b},\frac{-b}{1-b})$	1	1	$\frac{1}{1-b}[\rho_1 + \rho_2 - b(\rho_3 + \rho_4)]$	$1 \times 2 \times 2(1-b)$
	$(1,0;-1,1;0,-1)$	-1	1	$\rho_3 - \rho_4$	$2 \times 1 \times 2(1-b)$
	$(1,1;0,1;0,-1)$	-1	1	$\rho_3 - \rho_4$	$1 \times 2 \times 2(1-b)$
BBa/b	$(0,1;1,0;\frac{1}{1-b},\frac{-b}{1-b})$	0	1	$\frac{1}{1-b}[\rho_2 - b\rho_4]$	$2 \times 2 \times 2(1-b)$
	$(1,1;1,1;\frac{1}{1-b},\frac{-b}{1-b})$	0	2	$\frac{2}{1-b}[\rho_2 - b\rho_4]$	$1 \times 1 \times 2(1-b)$
	$(0,1;0,1;0,-1)$	-1	0	ρ_3	$2 \times 2 \times 2(1-b)$
	$(1,1;-1,1;0,-1)$	-2	0	$2\rho_3$	$1 \times 1 \times 2(1-b)$

Taking into account the sublety that, effectively, the O6-planes wrap the short cycles ρ_i while N_a D6-branes wrapping the long cycles ρ_i' support the gauge group $U(N_a)$, the RR tadpole cancellation condition takes the form

$$\begin{aligned}
\mathbf{AAa/b}:\ & \textstyle\sum_a N_a\left[Y_a n_3^a(\rho_1 - b\rho_3) + Z_a M_3^a \rho_4\right] = 12\left[(\rho_1 - b\rho_3) - (1-b)\rho_4\right], \\
\mathbf{ABa/b}:\ & \textstyle\sum_a N_a\left[(Y_a + Z_a)\, n_3^a\left(\rho_1 + \rho_2 - b(\rho_3 + \rho_4)\right) + (Y_a - Z_a)\, M_3^a(\rho_3 - \rho_4)\right] \\
& = 16\left[(\rho_1 + \rho_2) - b(\rho_3 + \rho_4) + (1-b)(\rho_3 - \rho_4)\right], \\
\mathbf{BBa/b}:\ & \textstyle\sum_a N_a\left[Z_a n_3^a(\rho_2 - b\rho_4) + Y_a M_3^a \rho_3\right] = 12\left[(\rho_2 - b\rho_4) + (1-b)\rho_3\right].
\end{aligned} \tag{16}$$

For the explicit computation of the RR tadpole cancellation conditions via open string 1-loop amplitudes and worldsheet duality see[25,33].

The resolution of the cycles in terms of $\mathcal{R}$ even and odd components is given in table 4 with $\Pi_i^+ \circ \Pi_j^- = -\frac{\kappa}{1-b}\delta_{ij}$ and $\kappa = 1$ for **AAa/b**, **BBa/b** and $\kappa = 2$ for **ABa/b**. The conclusion that two RR scalars participate in the generalized

Table 4. $\mathcal{R}$ even and odd cycles for $T^6/(\mathbb{Z}_4 \times \mathbb{Z}_2 \times \Omega\mathcal{R})$

lattice	Π_1^+	Π_2^+	Π_1^-	Π_2^-
AAa/b	$\frac{1}{1-b}[\rho_1 - b\rho_3]$	ρ_4	ρ_3	$-\frac{1}{1-b}[\rho_2 - b\rho_4]$
ABa/b	$\frac{1}{1-b}[\rho_1 + \rho_2 - b(\rho_3 + \rho_4)]$	$\rho_3 - \rho_4$	$\rho_3 + \rho_4$	$-\frac{1}{1-b}[\rho_1 - \rho_2 - b(\rho_3 - \rho_4)]$
BBa/b	$\frac{1}{1-b}[\rho_2 - b\rho_4]$	ρ_3	ρ_4	$-\frac{1}{1-b}[\rho_1 - b\rho_3]$

Green Schwarz mechanism and thus at most two $U(1)$ factors can acquire a mass is confirmed by the explicit calculation of the closed string spectrum displayed in table 5. The untwisted sector provides two real RR scalars. Any other RR sector contains either vectors or no massless states. In table 5, the bosonic degrees of freedom (d.o.f.) in terms of real scalars and vectors are listed. The fermionic d.o.f. follow from supersymmetry. It can be easily checked that the number of closed string chiral and vector multiplets is indeed $h_{1,1} + h_{2,1} = 62$.

Table 5. Closed string spectrum for $T^6/(\mathbb{Z}_4 \times \mathbb{Z}_2 \times \Omega\mathcal{R})$

untwisted	NSNS: Graviton + Dilaton + 7 scalars; RR: Axion + 1 scalar
$\theta + \theta^3$	NSNS: 16 scalars, RR: —
θ^2	**AAa/b, BBa/b:** NSNS: 20 scalars, RR: — **ABa/b:** NSNS: 18 scalars, RR: 1 vector
ω	**AAa, ABa, BBa:** NSNS: 24 s RR: — **AAb, ABb, BBb:** NSNS: 18 s RR: 3 v
$\theta\omega + \theta^3\omega$	**AAa, ABa, BBa:** NSNS: 32 s RR: — **AAb, ABb, BBb:** NSNS: 24 s RR: 4 v
$\theta^2\omega$	**AAa, ABa, BBa:** NSNS: 24 s RR: — **AAb, ABb, BBb:** NSNS: 18 s RR: 3 v

From the cycles (10), their $\mathcal{R}$ images (12), the supersymmetry condition (15), RR tadpole cancellation (16) and coefficients for the Green Schwarz couplings, any massless chiral spectrum and its physical $U(1)$ factors can be computed.

In the following section, some specific models are discussed.

2.2. *Chiral spectra: A no-go theorem for three generations*

The main goal of four dimensional string compactifications is to find stable vacua with the gauge group and chiral matter content of the standard model or, e.g. an $SU(5)$ or Pati-Salam GUT model. While engineering the required gauge group is comparatively easy, obtaining three quark generations is a complicated, in some scenarios even impossible task.

Three different phenomenologically appealing scenarios with a supersymmetric visible sector are conceivable in the framework of D6-branes intersecting at angles:

(i) The gauge group contains $SU(3) \times SU(2) \times U(1)$. All left- and right-handed quarks are realized as bifundamental representations. Pati-Salam and left-right symmetric models also belong to this category.

(ii) The gauge group contains $SU(3) \times SU(2)$. The left-handed quarks transform as bifundamentals, whereas the right-handed quarks are realized as antisymmetric representations of $SU(3)$.

(iii) The gauge group contains $SU(5) \times U(1)$. The left handed quarks sit in three antisymmetric representations of $SU(5)$, and three bifundamentals provide the remaining right-handed quarks.

The three scenarios have to fulfill the following minimal requirements in order to

provide stable[34,35] genuine three generation models:

- Supersymmetry of the visible sector.
- Three intersections of the QCD-stack with a second stack.
- No chiral fermion in the symmetric representation of the QCD-stack.

The intersection numbers of factorizable 3-cycles and their $\mathcal{R}$ images can be computed from (8), (10) and (12),

$$\Pi_a \circ \Pi_b = (Y_a Y_b + Z_a Z_b)\left(-n_3^a M_3^b + M_3^a n_3^b\right),$$

$$\Pi_a \circ \Pi_{b'} = \begin{cases} (Y_a Y_b - Z_a Z_b)\left(n_3^a M_3^b + M_3^a n_3^b\right) & \mathbf{AAa/b}, \\ (Y_a Z_b + Z_a Y_b)\left(n_3^a M_3^b + M_3^a n_3^b\right) & \mathbf{ABa/b}, \\ (-Y_a Y_b + Z_a Z_b)\left(n_3^a M_3^b + M_3^a n_3^b\right) & \mathbf{BBa/b}. \end{cases}$$

The net number of fundamental representations of $U(N_a)$ is therefore given by

$$\Pi_a \circ (\Pi_b + \Pi_{b'}) = \begin{cases} 2\left[Y_a Y_b M_3^a n_3^b - Z_a Z_b n_3^a M_3^b\right] & \mathbf{AAa/b}, \\ (Y_a + Z_a)(Y_b + Z_b) M_3^a n_3^b - (Y_a - Z_a)(Y_b - Z_b) n_3^a M_3^b & \mathbf{ABa/b}, \\ 2\left[-Y_a Y_b n_3^a M_3^b + Z_a Z_b M_3^a n_3^b\right] & \mathbf{BBa/b}. \end{cases} \quad (17)$$

For the untilted T_3^2, $b = 0$ implies that $M_3^c = m_3^c \in \mathbb{Z}$, and it is evident that the **AAa** and **BBa** orientations can only provide even numbers of fundamental representations of $U(N_a)$. This observation is independent of supersymmetry.

If stacks of D6$_b$-branes are realized on top the O6-planes, the gauge group is $Sp(2N_b)$, and the number of bifundamental representations is given by $\Pi_a \circ \Pi_b$. In table 3, the wrapping numbers for the different O6-planes are given. If the corresponding cycles are labeled by Π_{b1}, Π_{b2} for $n_3 \neq 0, M_3 = 0$ and Π_{c1}, Π_{c2} for $n_3 = 0, M_3 \neq 0$, the intersection numbers of an arbitrary factorizable cycle Π_a with the $\mathcal{R}$ invariant ones are

$$\Pi_a \circ \Pi_{bl} = \frac{1}{1-b} M_3^a \begin{cases} l Y_a, \\ (Y_a + Z_a), \\ l Z_a, \end{cases} \qquad \Pi_a \circ \Pi_{ck} = n_3^a \begin{cases} k Z_a & \mathbf{AAa/b}, \\ (Z_a - Y_a) & \mathbf{ABa/b}, \\ -k Y_a & \mathbf{BBa/b}. \end{cases} \quad (18)$$

For $k, l = 2$ and the **AAa/b**, **BBa/b** lattices, odd numbers of generations are not possible. As in the discussion of (17), the statement is independent of supersymmetry.

The number of (anti)symmetric chiral representations is computed from

$$\Pi_a \circ \Pi_{a'} = 2 n_3^a M_3^a \begin{cases} \left(Y_a^2 - Z_a^2\right) & \mathbf{AAa/b}, \\ 2 Y_a Z_a & \mathbf{ABa/b}, \\ \left(Z_a^2 - Y_a^2\right) & \mathbf{BBa/b}, \end{cases}$$

$$\Pi_a \circ \Pi_{O6} = 2 \begin{cases} 3\left[Y_a M_3^a + (1-b) Z_a n_3^a\right] & \mathbf{AAa/b}, \\ 2\left[(Y_a + Z_a) M_3^a - (1-b)(Y_a - Z_a) n_3^a\right] & \mathbf{ABa/b}, \\ 3\left[-(1-b) Y_a n_3^a + Z_a M_3^a\right] & \mathbf{BBa/b}, \end{cases} \quad (19)$$

and only for $\Pi_a \circ \Pi_{a'} = \Pi_a \circ \Pi_{O6}$, the spectrum is free of chiral symmetric states. By (15), at the special radii $r = \frac{1}{1-b}$ all supersymmetric 3-cycles fulfill $\Pi_a \circ \Pi_{O6} = 0$.

This means that the number of antisymmetric and symmetric representations are zero simultaneously and only scenario (i) is potentially accessible.

Formally, $\Pi_a \circ \Pi_{a'} = \Pi_a \circ \Pi_{O6}$, $\Pi_a \circ (\Pi_b + \Pi_{b'}) = \pm 3$ (or $\Pi_a \circ \Pi_{bl/ck} = \pm 3$ due to $SU(2) \equiv Sp(2)$) and the supersymmetry condition (15) can be solved for (n_3^a, m_3^a), (Y_a, Z_a), b and r. It turns out that the solutions require a square T_3^2, i.e. $r = \frac{1}{1-b}$, and e.g. on **BBA/B** $(n_3^a, m_3^a) = (1, -1)$, $(Y_a, Z_a) = (-3, 3)$ up to exchanging Π_a with its $\mathcal{R}$ image $\Pi_{a'}$. However, the expansion (11) in terms of wrapping numbers (n_k^a, m_k^a) which have to be coprime per T_k^2 $(k = 1, 2)$ does not have any solution to $|Y_a| = |Z_a| = 3$. The computation for the **AAa/b** and **ABa/b** orientations is analogous. In particular, all results for **AAa/b** are obtained from those of **BBa/b** by substituting $(Y_a, Z_a) \rightarrow (-Z_a, Y_a)$.

In summary, on the $T^6/(\mathbb{Z}_4 \times \mathbb{Z}_2 \times \Omega\mathcal{R})$ orientifold no genuine three generation models on factorizable 3-cycles fulfilling the requirements stated at the beginning of this section exist. In the next section, we present instead a 2+1 generation model.

2.3. *A 2+1 generation model with brane recombination*

In [33,36] a 2+1 generation model on the **ABB** lattice was presented which contains a breaking $SU(3)^2 \rightarrow SU(3)_{diag}$ and thereby converts the cycle which the QCD-stack wraps into a non-factorizable one. The $SU(2)_L \times SU(2)_R$ stacks remain factorizable, allowing for the computation of all possible Higgs-multiplet candidates from the non-chiral sector.

In this section, we present a similar construction on the **BBB** lattice. As in [33,36], two non-trivial kinds of D6$_{aj}$-branes with angles $(\tilde{\varphi}_1, \tilde{\varphi}_2, \tilde{\varphi}_3)\pi = (1/4, 0, -1/4)\pi$ and $(0, 1/4, -1/4)\pi$ w.r.t. the $\mathcal{R}$ invariant axis together with the D6$_{bl}$- and D6$_{ck}$-branes on top of some O6-planes are needed. The set-up is given in table 6. The resulting

Table 6. D6-brane configuration for a 2+1 generation model with $b = \frac{1}{2}$ and $r = 2$

D6-brane	N	$(n_1, m_1; n_2, m_2; n_3, m_3)$	Y	Z	cycle
a_1	3	(0,1;1,1;1, -1)	-1	1	$[-\rho_1 + \rho_2 - (-\rho_3 + \rho_4)]$
$a_2^{(i)}$, $i = 1, 2, 3$	1	(1,1;0,1;1, -1)	-1	1	$[-\rho_1 + \rho_2 - (-\rho_3 + \rho_4)]$
b_1	1	(0,1;1,0;2,-1)	0	1	$2\,[\rho_2 - b\rho_4]$
b_2	1	(1,1;1,1;2,-1)	0	2	$4\,[\rho_2 - b\rho_4]$
c_1	1	(0,1;0,1;0,-1)	-1	0	ρ_3
c_2	1	(1,1;-1,1;0,-1)	-2	0	$2\rho_3$

gauge group is

$$U(3)_{a1} \times (U(1)_{a2})^3 \times SU(2)_{b1} \times SU(2)_{b2} \times SU(2)_{c1} \times SU(2)_{c2} \tag{20}$$

with the identification $Sp(2) \equiv SU(2)$. The three stacks $a_2^{(i)}$ wrap the same cycles and are parallely displaced on some 2-torus, e.g. on T_3^2.

The massless $U(1)$ factors can be computed using (6) and table 4. It turns out, that apart from $B - L$, which occurs in all known left-right symmetric models

with D6-branes at angles, two additional $U(1)$ factors remain massless but without obvious physical interpretation,

$$\begin{aligned} Q_{B-L} &= \frac{1}{3} Q_{a1} - Q_{a2}^{(1)}, \\ Q' &= Q_{a2}^{(2)} - Q_{a2}^{(3)}, \\ Q'' &= \frac{3}{5} Q_{a1} + \frac{1}{5} Q_{a2}^{(1)} - Q_{a2}^{(2)} - Q_{a2}^{(3)}. \end{aligned} \tag{21}$$

The fourth $U(1)$ factor acquires a mass through its non-vanishing coupling to the B_2^i. The gauge group of the set-up with factorizable 3-cycles therefore is

$$SU(3)_{a1} \times \underbrace{SU(2)_{b1} \times SU(2)_{b2}}_{} \times \underbrace{SU(2)_{c1} \times SU(2)_{c2}}_{} \times U(1)_{B-L} \times U(1)' \times U(1)'' \times U(1)_m . \tag{22}$$

The chiral spectrum is given in table 7. In order to shorten the notation, a $\mathbf{3}_{-1}$ of

Table 7. Chiral spectrum of the 2+1 generation model

Sector	rep	$B-L$	particle	Sector	rep	$B-L$
$a_1 b_1$	$(\overline{\mathbf{3}}_{a1}, \mathbf{2}_{b1})$	-1/3	Q_R	$a_2^{(2)} b_1$	$(\overline{\mathbf{1}}_{a2^{(2)}}, \mathbf{2}_{b1})$	0
$a_1 b_2$	$2(\overline{\mathbf{3}}_{a1}, \mathbf{2}_{b2})$	-1/3	Q_R	$a_2^{(2)} b_2$	$2(\overline{\mathbf{1}}_{a2^{(2)}}, \mathbf{2}_{b2})$	0
$a_1 c_1$	$(\mathbf{3}_{a1}, \mathbf{2}_{c1})$	1/3	Q_L	$a_2^{(2)} c_1$	$(\mathbf{1}_{a2^{(2)}}, \mathbf{2}_{c1})$	0
$a_1 c_2$	$2(\mathbf{3}_{a1}, \mathbf{2}_{c2})$	1/3	Q_L	$a_2^{(2)} c_2$	$2(\mathbf{1}_{a2^{(2)}}, \mathbf{2}_{c2})$	0
$a_2^{(1)} b_1$	$(\overline{\mathbf{1}}_{a2^{(1)}}, \mathbf{2}_{b1})$	1	E_R, N_R	$a_2^{(3)} b_1$	$(\overline{\mathbf{1}}_{a2^{(3)}}, \mathbf{2}_{b1})$	0
$a_2^{(1)} b_2$	$2(\overline{\mathbf{1}}_{a2^{(1)}}, \mathbf{2}_{b2})$	1	E_R, N_R	$a_2^{(3)} b_2$	$2(\overline{\mathbf{1}}_{a2^{(3)}}, \mathbf{2}_{b2})$	0
$a_2^{(1)} c_1$	$(\mathbf{1}_{a2^{(1)}}, \mathbf{2}_{c1})$	-1	L	$a_2^{(3)} c_1$	$(\mathbf{1}_{a2^{(3)}}, \mathbf{2}_{c1})$	0
$a_2^{(1)} c_2$	$2(\mathbf{1}_{a2^{(1)}}, \mathbf{2}_{c2})$	-1	L	$a_2^{(3)} c_2$	$2(\mathbf{1}_{a2^{(3)}}, \mathbf{2}_{c2})$	0

$SU(3)_{a1} \times U(1)_{a1}$ is denoted as $\overline{\mathbf{3}}_{a1}$ and $\overline{\mathbf{1}}_{a2^{(1)}}$ denotes charge -1 under $U(1)_{a2}^{(1)}$. The particles on the left side of the table have the quantum numbers of the left-right symmetric standard model provided that two brane recombinations occur, namely $SU(2)_{b1} \times SU(2)_{b2} \to SU(2)_R$ and $SU(2)_{c1} \times SU(2)_{c2} \to SU(2)_L$. In both cases, b_1 and $(\Theta^k b_2)$ as well as c_1 and $(\Theta^k c_2)$ are parallel on T_3^2 and intersect once on $T_1^2 \times T_2^2$ for $k = 0, 1$ each. The intersections provide two hypermultiplets in $(\mathbf{2}_{b1}, \mathbf{2}_{b2})$ and $(\mathbf{2}_{c1}, \mathbf{2}_{c2})$ which are required for D- and F-flatness of the brane recombination process. Since the recombined branes still factorize into 2-cycles times 1-cycles on $T^4 \times T_3^2$, the recombined stack $B = b_1 + b_2$ can be displaced from the $\mathcal{R}$ invariant position on T_3^2. In this way, the electro weak breaking $SU(2)_R \times U(1)_{B-L} \to U(1)_Y \times U(1)'''$ can be realized.

The states listed on the right side of the table constitute additional vector-like matter.

A similar construction is possible on the **AAB** lattice as is immediately evident from the symmetry of the formulae under the exchange $(Y_a, Z_a) \to (-Z_a, Y_a)$.

In constrast to the previously presented model on the **ABB** lattice, the construction is not based on a Pati-Salam group $SU(4)$, and the non-factorizable branes

arise from the 'leptonic' and 'right' stacks. The lengths of the cycles Π_{a1} and Π_{a2}, however, are identical as in any Pati-Salam construction and thus fulfill a minimal requirement on a possible gauge unification.[37]

3. Genuine three generation models on isotropic tori

In section 2.2, it has been shown that on the $T^6/(\mathbb{Z}_4 \times \mathbb{Z}_2 \times \Omega\mathcal{R})$ orientifold, phenomenologically interesting supersymmetric genuine 3-generation models are not accessible. The analysis includes only the conditions for supersymmetry, no symmetric representations on the QCD-stack and three quark generations. If these conditions were fulfilled, the next requirement would be that the QCD-stack did not intersect with any of the remaining stacks of the model besides from those providing the left- and right-handed quarks. In the following, we briefly discuss how phenomenological model building with D6-branes at angles in other orbifold backgrounds is affected by these restrictions.

The remaining symmetric orbifolds which preserve $\mathcal{N}=1$ supersymmetry and have factorizable tori and 3-cycles fall into two classes. The $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$[38,25,39,40], $T^6/\mathbb{Z}_3$[24,41,42] and $T^6/(\mathbb{Z}_3 \times \mathbb{Z}_3)$[25] orbifolds have only bulk cycles and $\mathcal{N}=4$ gauge multiplets, whereas fractional 3-cycles and $\mathcal{N}=2$ gauge multiplets occur for $T^6/\mathbb{Z}_N$ $(N=4,6,6')$[24,43,44] and $T^6/(\mathbb{Z}_6 \times \mathbb{Z}_3)$[25,44].

3.1. *Orbifolds with factorizable O6-planes and bulk cycles only*

The $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ case has been studied in great detail. The $\mathbb{Z}_2$ actions do not constrain the ratio of radii on any 2-torus, and there is a rich variety of models which in general require anisotropic compact dimensions. However, also on this orbifold, there is not any known model fulfilling all requirements mentioned in section 2.2 plus vanishing intersections of the QCD-stack with any further stack, see e.g. [45,46,47], and the exotic chirals have to confine into composite matter fields due to a strongly coupled hidden sector[48] which can also lead to a dynamical supersymmetry breaking.[49]

The $T^6/\mathbb{Z}_3$ and $T^6/(\mathbb{Z}_3 \times \mathbb{Z}_3)$ orbifold are not suitable for supersymmetric model building since only two linearly independent bulk cycles exist. Supersymmetry projects onto one specific bulk cycle, and all intersection numbers among D6-branes and with the O6-planes vanish automatically. Therefore, no chiral fermions arise in supersymmetric set-ups.

3.2. *Orbifolds with factorizable O6-planes and fractional cycles*

For the $T^6/\mathbb{Z}_4$ case, the RR tadpole cancellation conditions and supersymmetric fractional cycles were computed and a 1+1+1 generation model has been presented [43]. The shape of the tori is identical to the $T^6/(\mathbb{Z}_4 \times \mathbb{Z}_2)$ case, but the exceptional cycles change the pattern of possible intersection numbers. The examination of this pattern shows that supersymmetric genuine three generation models are not accessible.

The $T^6/(\mathbb{Z}_6 \times \mathbb{Z}_3)$ orbifold has,[44] up to normalization, the same bulk 3-cycles as $T^6/\mathbb{Z}_3$ plus two exceptional 3-cycles stuck at the $\mathbb{Z}_2$ fixed points. The geometric interpretation of the loop channel amplitudes enforces a specific combination of the supersymmetric bulk 3-cycle with exceptional ones, not leaving enough freedom for phenomenological model building. The $T^6/\mathbb{Z}_6$ and $T^6/\mathbb{Z}_6'$ orbifolds have enough bulk and exceptional 3-cycles to be of phenomenological interest. While the latter case is currently under investigation[50], the former has already been examined systematically[44]. It turns out that in this background genuine three generation models exist which fulfill all the requirements mentioned above. As observed first for toroidal compactifications[51], the RR tadpole cancellation conditions are stronger than the conditions on vanishing gauge anomalies. Therefore, all models constructed in[44] have a non-minimal Higgs sector. In particular, one obtains exactly the chiral spectrum of the supersymmetric standard model with three Higgs generations and isotropic compact dimensions. In the remainder of this section, we briefly comment on the construction of this supersymmetric standard model on fractional branes in the $T^6/\mathbb{Z}_6$ case.

3.2.1. *Generalities on fractional branes*

In order to clarify the role of the exceptional cycles, let us first look at six dimensional IIB orientifolds on $\mathbb{R}^{1,5} \times T^4/(\mathbb{Z}_N \times \Omega\mathcal{R})$. The closed string spectrum of the IIB theory contains[52] in the k^{th} twisted sector a NSNS triplet of scalars $\vec{\chi}_k$ associated to the metric moduli (complex and Kähler deformations) and a NSNS scalar $b_k^{(0)} = \int_{e^{(k)}} {}^{(10)}B_2$ per fixed point. The RR sector contributes a RR scalar $\phi_k = \int_{e^{(k)}} {}^{(10)}C_2$ and a 2-form $C_2^k = \int_{e^{(k)}} {}^{(10)}C_4$ per fixed point. The fermionic superpartners arise from the R-NS and NS-R sectors.

The orientifold projection Ω identifies the k^{th} twisted sector with its inverse $(N-k)^{th}$ sector. For N odd, the counting of the twisted closed string states comprises the sectors $k = 1, \ldots, [N/2]$ and $k = [N/2]+1, \ldots, N-1$ contains no further physical d.o.f.. In terms of the Fourier transformed formulation, this means that exceptional cycles from inverse sectors are identified, $e^{(k)} \stackrel{\Omega}{\to} \pm e^{(N-k)}$. For N odd, an additional sign in the projection corresponds to a redefinition of the inverse cycle and does not affect the closed string spectrum. For $N = 2M$, the same reasoning applies to all sectors with $k \neq M$, whereas the contribution to the closed string spectrum from $\mathbb{Z}_2$ twisted sectors depends on the sign which the exceptional cycle picks up.

The general situation is different for the projection $\Omega\mathcal{R}$. In this case, the orientifold projection preserves each twist sector separately, $e^{(k)} \stackrel{\Omega\mathcal{R}}{\to} e^{(k)}$, and the explicit computation of the closed string spectra[53,41] shows that if the orbifold fixed point at which the cycle $e^{(k)}$ is located is also invariant under $\mathcal{R}$, one hypermultiplet per twist sector emerges, while orbifold fixed points which form pairs under $\mathcal{R}$ support a hyper and a tensor multiplet.

The O7-planes wrap the factorizable bulk cycles which are invariant under some

element $\Omega\mathcal{R}\mathbb{Z}_k$ of the orientifold group. However, in general not the bulk cycles but rather linear combinations of bulk and exceptional 2-cycles of the form

$$\Pi_a^{frac} = \frac{1}{2}\Pi_a^{bulk} + \frac{(-1)^{\tau_0}}{2}\left(e_{ik} + (-1)^{\tau_1} e_{jk} + (-1)^{\tau_2} e_{il} + (-1)^{\tau_1+\tau_2} e_{jl}\right) \tag{23}$$

provide an unimodular basis for the lattice on $T^4/\mathbb{Z}_2$ where $i, j \in T_1^2$ and $k, l \in T_2^2$ label the fixed points the bulk cycle passes through. $\tau_0 = 0, 1$ corresponds to the eigenvalues ± 1 of $\mathbb{Z}_2$, and $\tau_k = 0, 1$ denotes a discrete Wilson line on T_k^2 $(k = 1, 2)$. Any bulk cycle which passes through the orbifold fixed points can be decomposed into its fractional components, $\Pi_a^{bulk} = \frac{1}{2}\left(\Pi_a^{bulk} + \Pi_a^{ex}\right) + \frac{1}{2}\left(\Pi_a^{bulk} - \Pi_a^{ex}\right)$. While N_a D7-branes on a bulk cycle away from the orbifold point provide the gauge group $U(N_a)_{diag}$, on the fixed points (at the 'enhancon') each kind of N_a opposite fractional D7-branes provides the gauge group $U(N_a)$, i.e. $U(N_a)_{diag} \to U(N_a)^2$.[54] If furthermore the cycles are their own $\Omega\mathcal{R}$ images, each $U(N_a)$ is enhanced to $SO(2N_a)$ or $Sp(2N_a)$.

The general line of reasoning carries over to IIA models on $T^6/\mathbb{Z}_{2N}$ with D6-branes. In the $\mathbb{Z}_2$ subsector, the theory takes the local form $T^4/\mathbb{Z}_2 \times T^2$, and the IIA orientifold is obtained from the IIB theory in this section by applying one T-duality along the additional 2-torus. The exceptional 3-cycles decompose into exceptional 2-cycles times bulk 1-cycles, and the RR fields of the effective four dimensional theory are given in (3). The orbifold invariant bulk and exceptional cycles are computed along the same lines as for the $T^6/(\mathbb{Z}_4 \times \mathbb{Z}_2)$ case described in section 2.1, and the fractional cycles are computed in the spirit of (23).

3.2.2. *Three generations on the $T^6/\mathbb{Z}_6$ orbifold*

The $T^6/(\mathbb{Z}_6 \times \Omega\mathcal{R})$ model with fractional D6-branes at angles is the first known case with three quark generations and no additional chiral matter charged under the QCD-stack. In this construction, the replication of generations occurs via an intriguing interplay between discrete Wilson lines and the $\mathbb{Z}_3$ symmetry on the additional 2-torus, see figure 2. The observation that three families arise naturally from $\mathbb{Z}_3$ singularities is known from heterotic compactifications for nearly 20 years[55]. However, the discrete Wilson lines in the model with D6-branes at angles belong to the $\mathbb{Z}_2$ singularities of the orbifold symmetry $\mathbb{Z}_6$.

The details of the construction can be found in[44]. The chiral content of the supersymmetric standard model can be obtained from five stacks of D6-branes in a great variety of geometric settings. The chiral spectrum is insensitive to the specific realization and listed in table 8. The initial gauge group of the construction is $U(3)_a \times U(2)_b \times U(1)_c \times U(1)_d \times U(1)_e$, and after the generalized Green Schwarz mechanism, the anomaly-free group is

$$SU(3)_a \times SU(2)_b \times U(1)_{B-L} \times U(1)_c \times U(1)_e \times U(1)^2_{massive}. \tag{24}$$

The hypercharge is a linear combination of the massless Abelian groups, $Q_Y = \frac{1}{2}(Q_{B-L} + Q_c)$.

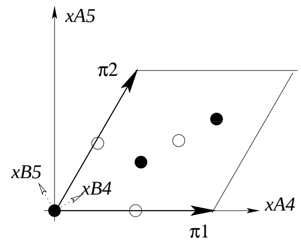

Fig. 2. Torus with $\mathbb{Z}_3$ invariance. Filled circles denote points fixed under $\mathbb{Z}_3$. In case of a $\mathbb{Z}_6$ symmetry, the empty circles are invariant under the $\mathbb{Z}_2$ subgroup. The origin is fixed under the complete orbifold group, whereas the fixed points of $\mathbb{Z}_2, \mathbb{Z}_3$ elements are permuted by the generator of $\mathbb{Z}_6$. As for the square tori of section 2.1, two orientations **A** and **B** are consistent with the involution $\mathcal{R}$.

Table 8. Chiral spectrum of 5 stack models on $T^6/(\mathbb{Z}_6 \times \Omega\mathcal{R})$

	sector	$SU(3)_a \times SU(2)_b$	Q_a	Q_b	Q_c	Q_d	Q_e	Q_{B-L}	Q_Y
Q_L	ab'	$3 \times (\overline{\mathbf{3}}, \mathbf{2})$	-1	-1	0	0	0	$\frac{1}{3}$	$\frac{1}{6}$
U_R	ac	$3 \times (\mathbf{3}, 1)$	1	0	-1	0	0	$-\frac{1}{3}$	$-\frac{2}{3}$
D_R	ac'	$3 \times (\mathbf{3}, 1)$	1	0	1	0	0	$-\frac{1}{3}$	$\frac{1}{3}$
L	bd'	$3 \times (1, \mathbf{2})$	0	1	0	1	0	-1	$-\frac{1}{2}$
E_R	cd	$3 \times (1, 1)$	0	0	1	-1	0	1	1
N_R	cd'	$3 \times (1, 1)$	0	0	-1	-1	0	1	0
	be	$3 \times (1, \mathbf{2})$	0	1	0	0	-1	0	0
	be'	$3 \times (1, \mathbf{2})$	0	1	0	0	1	0	0

The masses of the anomalous $U(1)$ factors and the non-chiral spectrum depend on the geometric details of the construction. In some cases, it is possible to recombine the stacks c and e such that the remaining two kinds of chiral multiplets obtain the quantum numbers of the Higgs particles. In this case, the Higgs sector is three times the one of the minimal supersymmetric standard model as expected from the 'anomaly' constraint on the $U(2)_b$ gauge factor.

The construction of left-right symmetric extensions of the standard model on $T^6/(\mathbb{Z}_6 \times \Omega\mathcal{R})$ follows the same lines, and again the Higgs sector is three times the minimal one.

The phenomenology of this class of models is currently under investigation.[50]

Acknowledgments

It is a pleasure to thank T. Ott for the collaboration on the $T^6/(\mathbb{Z}_6 \times \Omega\mathcal{R})$ models.[44] This work is supported by the RTN European program under contract number HPRN-CT-2000-00148.

1. M. Berkooz, M. R. Douglas and R. G. Leigh, *Nucl. Phys. B* **480**, 265 (1996)
2. R. Blumenhagen, L. Görlich, B. Körs and D. Lüst, *JHEP* **0010**, 006 (2000)

3. G. Aldazabal, S. Franco, L. E. Ibáñez, R. Rabadán and A. M. Uranga, *J. Math. Phys.* **42**, 3103 (2001)
4. G. Aldazabal, S. Franco, L. E. Ibáñez, R. Rabadán and A. M. Uranga, *JHEP* **0102**, 047 (2001)
5. A. M. Uranga, *Class. Quant. Grav.* **20**, S373 (2003)
6. D. Lüst, *Class. Quant. Grav.* **21**, S1399 (2004)
7. E. Kiritsis, *Fortsch. Phys.* **52**, 200 (2004)
8. C. Angelantonj and A. Sagnotti, *Phys. Rept.* **371**, 1 (2002) [Erratum-ibid. **376**, 339 (2003)]
9. C. Bachas, arXiv:hep-th/9503030.
10. C. Angelantonj, I. Antoniadis, E. Dudas and A. Sagnotti, *Phys. Lett. B* **489**, 223 (2000) [arXiv:hep-th/0007090].
11. C. Angelantonj and A. Sagnotti, arXiv:hep-th/0010279.
12. M. Larosa and G. Pradisi, *Nucl. Phys. B* **667**, 261 (2003) [arXiv:hep-th/0305224].
13. M. Cvetič, P. Langacker and G. Shiu, *Nucl. Phys. B* **642**, 139 (2002)
14. D. Cremades, L. E. Ibáñez and F. Marchesano, *JHEP* **0307**, 038 (2003)
15. M. Cvetič and I. Papadimitriou, *Phys. Rev. D* **68**, 046001 (2003)
16. S. A. Abel and A. W. Owen, *Nucl. Phys. B* **663**, 197 (2003)
17. D. Cremades, L. E. Ibáñez and F. Marchesano, arXiv:hep-th/0404229.
18. D. Lüst, P. Mayr, R. Richter and S. Stieberger, arXiv:hep-th/0404134.
19. S. A. Abel and A. W. Owen, *Nucl. Phys. B* **682**, 183 (2004)
20. I. R. Klebanov and E. Witten, *Nucl. Phys. B* **664**, 3 (2003)
21. S. A. Abel, M. Masip and J. Santiago, *JHEP* **0304**, 057 (2003)
22. S. A. Abel, O. Lebedev and J. Santiago, arXiv:hep-ph/0312157.
23. D. Lüst and S. Stieberger, arXiv:hep-th/0302221.
24. R. Blumenhagen, L. Görlich and B. Körs, *JHEP* **0001**, 040 (2000)
25. S. Förste, G. Honecker and R. Schreyer, *Nucl. Phys. B* **593**, 127 (2001)
26. R. Blumenhagen, V. Braun, B. Körs and D. Lüst, *JHEP* **0207**, 026 (2002)
27. R. Blumenhagen, J. P. Conlon and K. Suruliz, arXiv:hep-th/0404254.
28. D. Cremades, L. E. Ibáñez and F. Marchesano, *JHEP* **0207**, 022 (2002)
29. F. G. Marchesano Buznego, arXiv:hep-th/0307252.
30. T. Ott, *Fortsch. Phys.* **52**, 28 (2004)
31. R. Blumenhagen, B. Körs and D. Lüst, *JHEP* **0102**, 030 (2001)
32. M. Klein and R. Rabadán, *JHEP* **0010**, 049 (2000)
33. G. Honecker, *Nucl. Phys. B* **666**, 175 (2003)
34. R. Rabadán, *Nucl. Phys. B* **620**, 152 (2002)
35. D. Cremades, L. E. Ibáñez and F. Marchesano, *JHEP* **0207**, 009 (2002)
36. G. Honecker, arXiv:hep-th/0309158.
37. R. Blumenhagen, D. Lüst and S. Stieberger, *JHEP* **0307**, 036 (2003)
38. M. Berkooz and R. G. Leigh, *Nucl. Phys. B* **483**, 187 (1997)
39. M. Cvetič, G. Shiu and A. M. Uranga, *Phys. Rev. Lett.* **87**, 201801 (2001)
40. M. Cvetič, G. Shiu and A. M. Uranga, *Nucl. Phys. B* **615**, 3 (2001)
41. G. Pradisi, *Nucl. Phys. B* **575**, 134 (2000)
42. R. Blumenhagen, B. Körs, D. Lüst and T. Ott, *Nucl. Phys. B* **616**, 3 (2001)
43. R. Blumenhagen, L. Görlich and T. Ott, *JHEP* **0301**, 021 (2003)
44. G. Honecker and T. Ott, arXiv:hep-th/0404055.
45. M. Cvetič, I. Papadimitriou and G. Shiu, *Nucl. Phys. B* **659**, 193 (2003)
46. M. Cvetič and I. Papadimitriou, *Phys. Rev. D* **67**, 126006 (2003)
47. M. Cvetič, T. Li and T. Liu, arXiv:hep-th/0403061.
48. M. Cvetič, P. Langacker and G. Shiu, *Phys. Rev. D* **66**, 066004 (2002)

49. M. Cvetič, P. Langacker and J. Wang, *Phys. Rev. D* **68**, 046002 (2003)
50. G. Honecker, T. Ott, *work in progress*
51. L. E. Ibáñez, F. Marchesano and R. Rabadán, *JHEP* **0111**, 002 (2001)
52. M. R. Douglas and G. W. Moore, arXiv:hep-th/9603167.
53. R. Blumenhagen, L. Görlich and B. Körs, *Nucl. Phys. B* **569**, 209 (2000)
54. *For a review see e.g.* M. Bertolini, *Int. J. Mod. Phys. A* **18**, 5647 (2003)
55. L. E. Ibáñez, J. E. Kim, H. P. Nilles and F. Quevedo, *Phys. Lett. B* **191**, 282 (1987).